$\rm NMA$ – homework from week 1

1. Consider a matrix A and a vector v:

$$A = \begin{bmatrix} 4 & 2 & 5 \\ 0 & 3 & 1 \\ 0 & -13 & -1 \end{bmatrix} \qquad v = \begin{bmatrix} -6 \\ 4 \\ -1 \end{bmatrix}$$

a) Compute row, column and Frobenius norms of the matrix A and the vector v.

b) Compute spectral radius $\rho(A)$ of the matrix A and compare it with the norms computed before.

c) What can you now tell about a value of spectral norm of the matrix A? Check your forecast by computing the spectral norm (using Matlab).

2. Consider a matrix *A*:

$$A = \left[\begin{array}{rrr} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{array} \right]$$

a) Compute row, column and Frobenius norms of the matrix A.

b) Name some important properties of the matrix A and prove them.

c) Which of the four values $\{1, 0, -1, 3-2i\}$ cannot be eigenvalues of the matrix A and why? Compute all eigenvalues λ_i of the matrix A.

d) Compute spectral radius $\rho(A)$ of the matrix A and compare it with other norms computed before. What can you tell about spectral norm of this matrix, if you know its spectral radius?